Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐ **Total No. of Pages : 03**

Total No. of Questions : 09

B.Tech. (2011 Onwards) (Sem.–2)

ENGINEERING MATHEMATICS – II

Subject Code : BTAM-102

Paper ID : [A1111]

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C. have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**

SECTION A

1. Write briefly :

a) Define Linear independent and dependent vectors.

b) Find the characteristic equation of the matrix $\begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}$.

c) Test whether the matrix $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{pmatrix}$ is diagonalisable or not?

d) Express $\sin^5 \theta$ in terms of the sines of multiples of θ.

e) Find the general and principle value of i^i.

f) Examine the convergence / divergence of the series $\sum_{n=1}^{\infty} \frac{1}{(1+\frac{1}{n})^{n^2}}$.

g) Define the term absolute convergence and use this concept to test the convergence of the series $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$.

h) Find the degree and order of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \frac{d^2y}{dx^2}$

i) For what value of k the equation $xy^3dx + k(x^2 y^2)dy = 0$ is a exact equation.

j) Find the complete solution of the equation :

$$\frac{d^2x}{dt^2}+5\frac{dx}{dt}+6x=0, \text{ given } x(0) = 0, \frac{dx}{dt}(\text{at } t=0)=15.$$

SECTION-B

2. a) Use method of variation of parameters to find the general solution of the differential equation $y'' + y = \operatorname{cosec} x$.

   b) Find the complete solution of the differential $y''-2y'+y=x\sin x$.

3. a) Solve the following simultaneous differential equation

$$\frac{dx}{dt}+y+5x=e^t, \frac{dy}{dt}+x+5y=e^{5t}$$

   b) Find the complete solution of the differential equation $x^2y''-3xy'+5y=x\log x$ by using operator method.

4. a) Solve the differential equation : $\frac{dy}{dx}=\frac{y+1}{(y+2)e^x-x}$.

   b) Find the particular solution of the differential equation $y'' - 2y' + 2y = e^x \tan x$

5. An electric circuit consists of an inductance 0.1 henry, a resistance of 20 ohms, and a condenser of capacitance 25 microfarads. Find the charge q and the current i at time t, given the initial conditions $q = 0.05$ coulombs, $i = 0$ when $t = 0$.

SECTION-C

6. a) Use the rank method to test the consistency of the system of equations

   $x + 2y + z = 2;\ 3x + y - 2z = 1;\ 4x - 3y - z = 3;\ 2x + 4y + 2z = 4,$ if consistent, then solve it completely.

   b) Use Gauss-Jordan method to find the inverse of the matrix

$$\begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 5 \end{pmatrix}$$

7. a) Verify Cayley-Hamilton theorem for the matrix

$$\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$

   and hence find its inverse.

   b) Prove that eigen values of Hermetian matrix are real.

8. a) Test the convergence of the series

$$\frac{x}{1.2} + \frac{x^2}{3.4} + \frac{x^3}{5.6} + \ldots\ldots\ldots\infty (x > 0)$$

   b) Test for what values of x does the series convergence/diverge?

$$\sum_{n=1}^{\infty} \frac{(n+1)^n}{n^{n+1}} x^n$$

9. a) Use Demoivre's theorem to prove that

$$\left(\frac{1+\sin\alpha + i\cos\alpha}{1+\sin\alpha - i\cos\alpha}\right)^n = \cos\left(\frac{n\pi}{2} - n\alpha\right) + i\sin\left(\frac{n\pi}{2} - n\alpha\right)$$

   b) Separate $\tan^{-1}(e^{i\theta})$ into real and imaginary parts.