Roll No. Total No. of Pages : 03

Total No. of Questions : 09

B.Tech. (2011 Onwards) (Sem.–1)

ENGINEERING MATHEMATICS – I

Subject Code : BTAM-101

Paper ID : [A1101]

Time : 3 Hrs. Max. Marks : 60

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C. have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**
5. **Symbols used have their usual meanings. Statistical tables, if demanded, may be provided.**

SECTION-A

1. Solve the following :

a) If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, then show that $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$.

b) Find the stationary points of the function $f(x, y) = x^3 + y^3 - 63(x + y) + 12xy$.

c) If $u = x^2 - y^2$ and $v = 2xy$ and $x = r\cos\theta$, $y = r\sin\theta$, then find the value of $\frac{\partial(u,v)}{\partial(r,\theta)}$.

d) For what values of a, b, and c the vector function

$\vec{F} = (x + y + az)\vec{i} + (bx + 3y - z)\vec{j} + (3x + cy + z)\vec{k}$ is irrotational.

e) Calculate the circulation of the field $\vec{F} = (x - y)\hat{i} + x\hat{j}$ around the circle $x^2 + y^2 = 1$.

f) Find the length of one arc of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$.

g) Evaluate $\int_0^{2\sqrt{\ln 3}}\int_{y/2}^{\sqrt{\ln 3}} e^{x^2}\,dxdy$.

h) State Stoke's theorem.

i) Evaluate $\int_C xydx + (x + y)dy$, along the curve C : $y = x^2$ from (–1,1) to (2,4).

j) Evaluate : $\int_0^a\int_0^x\int_0^{x+y} e^{x+y+z}\,dz\,dy\,dx$.

SECTION-B

2. a) Find the radius of curvature at any point of the curve $r = a(1 - \cos\theta)$ and prove that ρ^2/r is constant.

   b) Trace the curve $y^2(x^2 + y^2) + a^2(x^2 - y^2) = 0$ by giving all its features in detail.

3. a) Find the volume of the solid formed by revolving the curve $y^2(2a - x) = x^3$ about its asymptote.

   b) Find the centre of gravity of the arc of the curve $x = a(t + \sin t)$, $y = a(1 - \cos t)$ in the first quadrant.

4. a) If $u = f(r)$, where $r^2 = x^2 + y^2$, then show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$

   b) Use Lagrange's method of undetermined coefficients to show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube.

5. a) Find the first three terms of the Taylor's series expansion of $e^x \log(1 + y)$ in the neighbourhood of (0,0)

   b) Use Euler's theorem to prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} = y^2 \frac{\partial^2 u}{\partial y^2} = \frac{\sin u \cos 2u}{4\cos^3 u}, \text{ whenever } u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$$

SECTION-C

6. a) Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} (x^2 + y^2)\, dy\, dx$ by changing into polar coordinates.

   b) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$.

7. a) Prove the identity :

$$\nabla\left[\frac{\vec{a} \cdot \vec{r}}{r^n}\right] = \frac{\vec{a}}{r^n} - \frac{n(\vec{a} \cdot \vec{r})\,\vec{r}}{r^{n+2}}, \text{ where } \vec{a} \text{ is a constant vector.}$$

b) A vector field is given by $\vec{F} = 18z\hat{i} - 12\hat{j} + 3y\hat{k}$. Evaluate the surface integral $\int_S \vec{F} \cdot \hat{n} ds$, where S is the part of the plane $2x + 3y + 6z = 12$ in the first octant.

8. Verify the Gauss Divergence theorem for a vector field defined by $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - xz)\hat{j} + (z^2 - xy)\hat{k}$ taken around the rectangular parallelepiped $0 \le x \le a,\ 0 \le y \le b,\ 0 \le z \le c$.

9. a) Find the directional derivative of $f(x, y.z) = x y^2 + y z^3$ at $(2,-1,1)$ in the direction of normal to the surface $x \log z - y^2 = -4$ at $(-1,2,1)$.

b) State Green's theorem in plane and use it to evaluate

$$\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy,$$

where C is the boundary of the region defined by $x = 0$, $y = 0$, $x + y = 1$